RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College affiliated to University of Calcutta)

FIRST YEAR [BATCH 2017-20]
B.A./B.Sc. SECOND SEMESTER (January – June) 2018
Mid-Semester Examination, March 2018

Date : 14/03/2018 **MATHEMATICS (Honours)**

Time : 11 am – 1 pm Paper : II Full Marks : 50

[Use a separate Answer Book for each group]

Group – A [12 marks]

1. Answer **any three** questions : [3×4]

a) Find the remainder when $x^4-3x^3+2x^2+x-1$ is divided by x^2-4x+3.

b) Let f(x) be a polynomial with real co-efficients having a root $\alpha+i\beta$ where α,β are real. Show that $\alpha-i\beta$ is also a root of f(x).

c) State Descartes' rule of signs. Apply this rule to find the nature of the roots of the equation $x^4+2x^2+3x-1=0$.

d) Solve : $x^4-2x^3+4x^2+6x-21=0$, given that the sum of two of the roots is zero.

e) Solve : $x^3+6x^2+11x+6=0$, given that the roots are in arithmetic progression.

Group – B [13 marks]

2. Answer **any one** question : [1×4]

a) Let $f:(0,1]\to\mathbb{R}$ be defined by

$$f(x)=\begin{cases}0, \text{ if } x\in(0,1]\cap\mathbb{Q}^C\\ \frac{1}{q}, \text{if } x\in(0,1]\cap\mathbb{Q} \text{ with } x=\frac{p}{q},\ p,q\in\mathbb{Z}, q>0 \text{ and gcd } (p,q)=1\end{cases}$$

Find all the points of continuity of f.

b) Let $f:[0,1]\to\mathbb{R}$ be a continuous map such that $f(0)<0$ and $f(1)>0$. Show that $\exists$ a point $c\in(0,1)$ such that $f(c)=0$.

3. Answer **any three** questions : [3×3]

a) If $\sum u_n$ be a convergent series of positive real numbers and $\{u_n\}$ is a monotone decreasing sequence. Prove that $\lim\limits_{n\to\infty} nu_n=0$. Is the converse of the above statement true? Discuss it with the help of a example. [2+1]

b) Examine the convergence of the series $\sum a_n$ where $a_n=n^p\left\{\frac{1}{\sqrt{n-1}}-\frac{1}{\sqrt{n}}\right\}$ for $n>1$. [3]

c) i) Prove that the following series is convergent : $1-\frac{1}{2}\left(1+\frac{1}{3}\right)+\frac{1}{3}\left(1+\frac{1}{3}+\frac{1}{5}\right)-....$ [2]

ii) If $\{a_n\}$ be a sequence such that $\lim\limits_{n\to\infty}(n^2a_n)$ exists in $\mathbb{R}$. Show that $\sum a_n$ is absolutely convergent. [1]

d) Let $f:\mathbb{R}\to\mathbb{R}$ be defined by $f(x)=x^5+4x+1,\ x\in\mathbb{R}$

i) Show that f has an inverse function g which is differentiable on $\mathbb{R}$. [1]

ii) Find $g'(1),\ g'(6)$. [2]

e) Let, $I=[a,b]$ and $f:I\to\mathbb{R}$ be differentiable on I. If $f'(a)\cdot f'(b)<0$, prove that there exist a point $C\in(a,b)$ such that $f'(C)=0$. [3]

Group – C [10 marks]

4. If A be a skew symmetric matrix of order n and P be an n×1 matrix, prove that $P^tAP=0$. [2]

5. Answer **any two** questions : [2×4]

a) If $A=\begin{pmatrix}0&1&2\\2&0&1\\1&2&0\end{pmatrix}$, show that $A^3-6A-9I_3=0$. Hence obtain a matrix B such that $BA=I_3$.

b) Show that $\begin{vmatrix}1+a_1&1&1&1\\1&1+a_2&1&1\\1&1&1+a_3&1\\1&1&1&1+a_4\end{vmatrix}=a_1a_2a_3a_4\left(1+\frac{1}{a_1}+\frac{1}{a_2}+\frac{1}{a_3}+\frac{1}{a_4}\right)$.

c) Find the matrix A if adj $A=\begin{pmatrix}2&2&0\\2&5&1\\0&1&1\end{pmatrix}$ and det A = 2.

Group – D [15 marks]

(*Answer any one question*) [1×15]

6. a) If the L.P.P Maximize z = cx, $x\geq 0$ subject to Ax = b admits of an optimal solution then prove that the optimal solution will coincide with at least one B.F.S. [7]

b) $x_1=1,\ x_2=1,\ x_3=1$ and $x_4=0$ is a F.S of the system of equations

$$x_1+2x_2+4x_3+x_4=7$$
$$2x_1-x_2+3x_3-2x_4=4$$

Reduce the F.S. to two different B.F.S. [4]

c) Prove that the set of all convex combinations of a finite number of points is a convex set. [4]

7. a) Use Big–M method to maximize $z=6x_1+4x_2$ subject to $2x_1+3x_2\leq 30,\ 3x_1+2x_2\leq 24$, $x_1+x_2\geq 3,\ x_1\geq 0,\ x_2\geq 0$. Show that the solution is not unique. Find two solutions. [7]

b) Prove that every extreme point of the convex set of all feasible solutions of Ax = b, $x\geq 0$ corresponds to a B.F.S. [5]

c) Find all the basic solutions of the equations

$$x_1+x_2+x_3=4$$
$$2x_1+5x_2-2x_3=3$$ [3]

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